Two-dimensional frustrated Ising model with four phases

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In this paper we consider a $d=2$ random Ising system on a square lattice with nearest neighbor interactions. The disorder is short range correlated and asymmetry between the vertical and the horizontal direction is admitted. More precisely, the vertical bonds are supposed to be nonrandom while the horizontal bonds alternate: one row of all nonrandom horizontal bonds is followed by one row where they are independent dichotomic random variables. We solve the model using an approximate approach that replaces the quenched average with an annealed average under the constraint that the number of frustrated plaquettes is kept fixed and equals that of the true system. The surprising fact is that for some choices of the parameters of the model there are three second-order phase transitions separating four different phases: antiferromagnetic, reentrant paramagnetic (glassy?), ferromagnetic, and paramagnetic. [S1063-651X(97)08709-6]

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I. INTRODUCTION

Mean field spin glass models have been studied and deeply understood both from a static and a dynamic point of view and key words like replica symmetry breaking, aging, and ultrametricity have become very widely used in statistical mechanics of disordered systems [1–5]. The reason spin glasses have attracted so much attention is probably more a consequence of the many successful applications to biological modeling (neural networks, immune system, adaptive evolution) than their original scope, which was limited to the description of disordered materials. For this reason and maybe for objective technical difficulties most of the typical features which are very well established for the mean field models have not been discovered for short range spin glasses. For example, it is commonly believed that a finite temperature glassy phase only exists for $d\geqslant 3$ spin glasses while in $d=2$ one has only the paramagnetic phase. This is an almost surely true statement if one considers a $d=2$ spin system with independent bonds [6–11] and with vertical-horizontal symmetry but it may be a false statement if one considers $d=2$ spin asymmetric systems with correlated disorder. For example, in models with layered disorder the existence of a low temperature phase seems to be an established fact [12–14]; nevertheless, one may think that these models are pathological since layered disorder is somehow a long range correlated disorder.

In this paper we consider a $d=2$ Ising system where there is both a short range correlation of the disorder and an asymmetry between vertical and horizontal direction. The specific interaction we chose is not motivated by a deep physical insight but it is merely dictated by technical reasons. Nevertheless, the model is not very artificial and the disorder correlation is limited to the fact that frustrated plaquettes always are present in near couples while the asymmetry only lies in a difference of strength of vertical and horizontal bonds.

We solve the model using an approximate approach that replaces the quenched average with an annealed average under the constraint that the number of frustrated plaquettes is kept fixed and equals that of the true system. The surprising fact is that for some choices of the parameters of the models one can find four different phases.

The paper is organized as follows. In Sec. II, after a brief overview of the constrained annealing, we introduce our model with particular attention to the concept of frustration; then we write the partition function with constrained frustration and the relative free energy. In Sec. III, we derive the solution of the model, obtaining an expression for the free energy that can be computed via numerical methods. Moreover, the exact ground state energy is found. In Sec. IV the conditions that yield to second-order phase transitions are derived. In Sec. V we describe the various behaviors of the model, showing a total of four distinct phases, three almost conventional (high temperature paramagnetic phase, ferromagnetic and antiferromagnetic phases at low temperature) and a reentrant paramagnetic phase that we guess to have a "glassy" nature. In Sec. VI we present our conclusions.

II. CONSTRAINED ANNEALING

The model is defined on a square $d=2$ lattice and the interaction is supposed to be effective only between nearest neighbors. The number of spins is $N=LM$ where M is the number of columns of the lattice and L is the number of rows.

The vertical bonds are supposed to be nonrandom and one can assume without loss of generality that they equal 1 while the horizontal bonds alternate; one row of all nonrandom horizontal bonds equal to 1 is followed by one row where they are independent dichotomic random variables which equal 1 with probability p and equal the negative value $-\gamma$ with probability $1-p$ (see Fig. 1).

It follows that the Hamiltonian of our model can be written as

$$H_N=-\sum_{i=1}^{L}\sum_{j=1}^{M}(\sigma_{i,j}\sigma_{i+1,j}+J_{i,j}\sigma_{i,j}\sigma_{i,j+1}), \qquad (2.1)$$

where $\sigma_{i,j}=\pm 1$ is the spin in the site located by the ith row and the jth column while the $J_{i,j}$ are the horizontal bonds which equal 1 when i is even and are defined by

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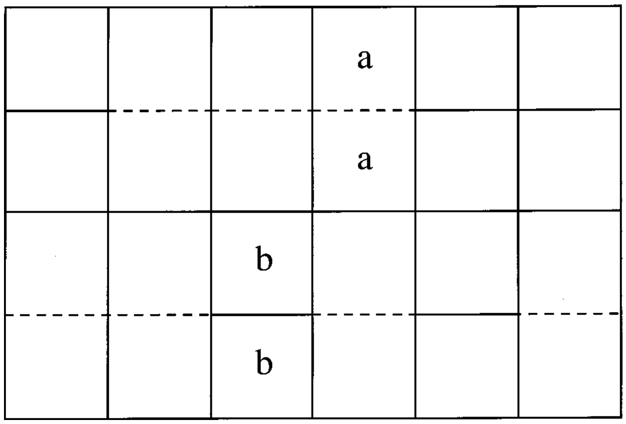

FIG. 1. A typical realization of the system. The full lines represent the $+1$ ferromagnetic bonds, while the dashed lines are the $-\gamma$ antiferromagnetic bonds. The a elementary plaquettes are frustrated, at difference with the b plaquettes.

$$J_{i,j}=\begin{cases}1 & \text{with probability } 1-p\\ -\gamma & \text{with probability } p\end{cases} \tag{2.2}$$

when i is odd.

The model is parametrized by $\gamma>0$ and p and, in general, it is random, except in the two limit cases $p\to 0$ and $p\to 1$. In the first limit case $p=0$ all the couplings equal unity and therefore we have the pure $d=2$ Ising model [15]. In the second limit case $p=1$ the model is also not random, but while all the vertical couplings equal unity, the horizontal couplings alternate one row in which they are all positive and equal to unity to one row in which they are all negative and equal to $-\gamma$. In this second limit the model can be solved by standard transfer matrix methods [15] and it shows a low temperature magnetic phase; for $\gamma<1$ this low temperature phase is ferromagnetic while for $\gamma>1$ there is horizontal antiferromagnetic order and vertical ferromagnetic order between the spins. For the sake of simplicity we will hereafter call this complicated magnetic phase simply the antiferromagnetic phase. Finally, the special choice $p=1$, $\gamma=1$ corresponds to the so-called ''fully frustrated model'' [16] which is also not random and has a transition only at $T=0$.

In order to explain the nature of our approximation, let us first recall that the elementary unit for frustration is the plaquette. If the product of the signs of the bonds around a plaquette is negative the plaquette is frustrated, otherwise the plaquette is unfrustrated. In our Ising model, only the sign of the random variable $J_{i,j}$ with odd i can be negative, therefore the two square plaquettes which share this bond are frustrated if this bond is effectively negative (e.g., a plaquettes in Fig. 1) and they are unfrustrated if it is positive (b plaquettes). As a consequence of this definition of elementary frustration, we may define the total frustration of the system ϕN as the rate of frustrated plaquettes. In our model

$$\phi_N=\frac{2}{N}\sum_{i=1}^{L}\sum_{j=1}^{M}\frac{1-J_{i,j}}{1+\gamma}. \tag{2.3}$$

This quantity equals p on average, furthermore, the strong law of large numbers assures that $\phi N\to p$ with probability 1 in the thermodynamic limit.

We are far from being able to solve the quenched model, nevertheless we think that the qualitative behavior of the system is captured by the above definition of total frustration ([17], for a more general definition see [18]). Therefore our proposal is to consider an annealed approximation where ϕ_N is constrained to coincide, in the thermodynamic limit, with the quenched total frustration p. This model corresponds to averaging Z only over the realizations of the disorder with total frustration p. We not only believe that the approximated model has the same qualitative features as the quenched one, but it is also in good quantitative agreement with it. In fact, our experience is that constrained annealing is a really powerful tool for estimating the free energy of disordered systems [19–24]. We would also like to stress that the fixed frustration model can also be seen as an independent model where the bonds as well as the spins are allowed to arrange themselves in order to minimize the free energy provided they satisfy the global frustration constraint.

In order to obtain the free energy of the fixed frustration model we follow the general method ([19,22]). We must first define the generalized partition function

$$Z_N(\beta,\gamma,\mu)=\sum_{\sigma}\overline{\exp[-\beta H_N+\mu N(\phi_N-p)]}, \tag{2.4}$$

where $\beta=1/T$ is the inverse temperature, and the average is over all realizations of the couplings $J_{i,j}$. Then we obtain the free energy of the constrained annealed model as

$$f(\beta,\gamma)=-\min_{\mu}\lim_{N\to\infty}\frac{1}{N\beta}\ln Z(\beta,\gamma,\mu), \tag{2.5}$$

where the $N\to\infty$ limit means that both M and L must tend to the same limit. In fact, the minimization over μ automatically selects the realizations of the disorder for which $\phi_N=p$ in the thermodynamic limit.

III. SOLUTION OF THE MODEL

The generalized partition function is a sum of a product of randomly independent variables, therefore we can write

$$Z_N(\beta,\gamma,\mu)=\sum_{\sigma}\prod_{i=1}^{L}\prod_{j=1}^{M}\overline{\exp\left[\beta\sigma_{i,j}\sigma_{i+1,j}+\beta J_{i,j}\sigma_{i,j}\sigma_{i,j+1}+\mu\left(2\frac{1-J_{i,j}}{1+\gamma}-p\right)\right]}.$$

The average can now be easily performed, obtaining

$$Z_N(\beta,\gamma,\mu)=\exp\left[\frac{N}{4}\{\ln[4p(1-p)]-2(2p-1)\mu+a\}\right]\times\sum_{\{\sigma\}}\exp[-\beta\widetilde{H}_N], \tag{3.1}$$

where we have introduced the effective Hamiltonian

$$\widetilde{H}_N=-\sum_{i=1}^{L}\sum_{j=1}^{M}(\sigma_{i,j}\sigma_{i+1,j}+\widetilde{J}_i\sigma_{i,j}\sigma_{i,j+1}).$$

The new effective horizontal bonds $\widetilde{J}_i$ are not random, they are all equal in the same row, and they alternate two possible values in different rows; in fact, one has $\widetilde{J}_i=1$ when i is even and $\widetilde{J}_i=b/2\beta$ when i is odd. The constants a and b are

$$a=\ln\left[\cosh\left(\beta\frac{1+\gamma}{2}-\mu+\frac{1}{2}\ln\frac{1-p}{p}\right)\times\cosh\left(\beta\frac{1+\gamma}{2}+\mu-\frac{1}{2}\ln\frac{1-p}{p}\right)\right], \tag{3.2}$$

$$b=\ln\frac{\cosh\left(\beta\frac{1+\gamma}{2}-\mu+\frac{1}{2}\ln\frac{1-p}{p}\right)}{\cosh\left(\beta\frac{1+\gamma}{2}+\mu-\frac{1}{2}\ln\frac{1-p}{p}\right)}+\beta(1-\gamma).$$

It is possible to show that b is a monotonic decreasing function of μ with $-2\gamma\beta\leqslant b\leqslant 2\beta$, so that we can directly use b as a variational parameter in order to realize the minimum in Eq. (2.5).

The effective Hamiltonian $\widetilde{H}_N$ is indeed associated to a pure $2d$ Ising model with unitary strength couplings along the vertical bonds, and with alternated rows of unitary and $b/2\beta$ strength couplings. This model can be solved by trivially generalizing the Onsager solution and it is mapped into the problem of diagonalizing a collection of 2×2 matrices. In the thermodynamic limit $N\rightarrow\infty$ the total free energy (2.5) reads

$$f(\beta,\gamma)=-\frac{\gamma+p(1-\gamma)}{2}-\frac{1}{2\beta}\ln\{4p^p(1-p)^{1-p}\times\sinh[\beta(1+\gamma)]\sinh(2\beta)\}-\min_b\left[\frac{b}{4\beta}(1-2p)-\frac{1-p}{2\beta}\ln(e^{b+2\beta\gamma}-1)-\frac{p}{2\beta}\ln(e^{2\beta-b}-1)+\frac{1}{4\pi\beta}\int_0^\pi dq\,\ln\lambda(q,b)\right], \tag{3.3}$$

where $\lambda(q,b)$ indicates the maximum eigenvalue in modulus of the product of the two matrices

$$T_\beta(q)=\exp[\beta^*(\tau_z\cos q+\tau_x\sin q)]\exp(-2\beta\tau_z),$$

$$\widetilde{T}_b(q)=\exp[\beta^*(\tau_z\cos q+\tau_x\sin q)]\exp(-b\tau_z),$$

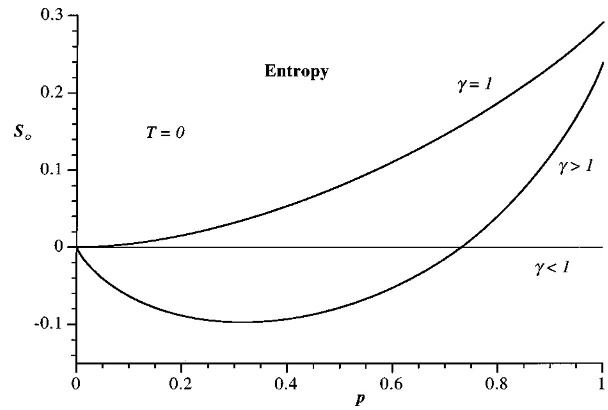

FIG. 2. Zero temperature entropy S_0 as a function of p at $\gamma<1$, $\gamma=1$, and $\gamma>1$. In the last case S_0 becomes positive for $p>\widetilde{p}$, where $\widetilde{p}<\frac{3}{4}$, and $S_0(p=\frac{3}{4})\simeq 0.01$.

where $\beta^*=-\ln\tanh\beta$, and τ_x,τ_z are Pauli matrices. After some trivial algebra one gets

$$\lambda(q,b)=t(q,b)+\sqrt{t(q,b)^2-1}, \tag{3.4}$$

with

$$t(q,b)=2\cos^2 q\,\frac{\sinh b}{\sinh 2\beta}-2\cos q\,\cosh 2\beta\,\frac{\sinh(2\beta+b)}{\sinh^2 2\beta}+\frac{\cosh^2 2\beta\,\cosh(2\beta+b)+\cosh(2\beta-b)}{\sinh^2 2\beta}. \tag{3.5}$$

The minimum of Eq. (3.3) is realized for $b=b^*$ and it is achieved by looking for the zero of its derivative. One has the self-consistent equation for b:

$$\left[1+\frac{2p}{e^{2\beta-b}-1}-\frac{2(1-p)}{1-e^{-2\beta\gamma-b}}+\frac{1}{4\pi}\int_0^\pi dq\,\frac{\partial t(q,b)/\partial b}{\sqrt{t(q,b)^2-1}}\right]_{b=b^*}=0. \tag{3.6}$$

When $p\rightarrow 0$ the model has to reduce to the standard Ising model and in fact, the previous formula leads to $b^*\rightarrow 2\beta$, while in the other limit case $p\rightarrow 1$ one has $b^*\rightarrow -2\gamma\beta$.

Equation (3.6) is an ordinary equation in b, nevertheless it cannot be explicitly solved, so we are not able to give a compact expression for b^* in terms of T, γ, and p. However, Eq. (3.6) and then Eq. (3.3) can be numerically computed with the necessary precision in order to fully investigate the model. Furthermore, at $T=0$ while we do not have the complete solution of Eq. (3.6) we are able to derive the leading terms of b^* and to compute the ground state energy U_0. We find out that U_0 has different linear behaviors in p depending on γ,

$$U_0=-2+\left(1-\frac{|1-\gamma|}{2}\right)p.$$

The $T=0$ entropy S_0 can only be computed numerically and it is shown in Fig. 2. Unlike the energy U_0, S_0 equals a constant function of p for any of the three choices of γ. It is

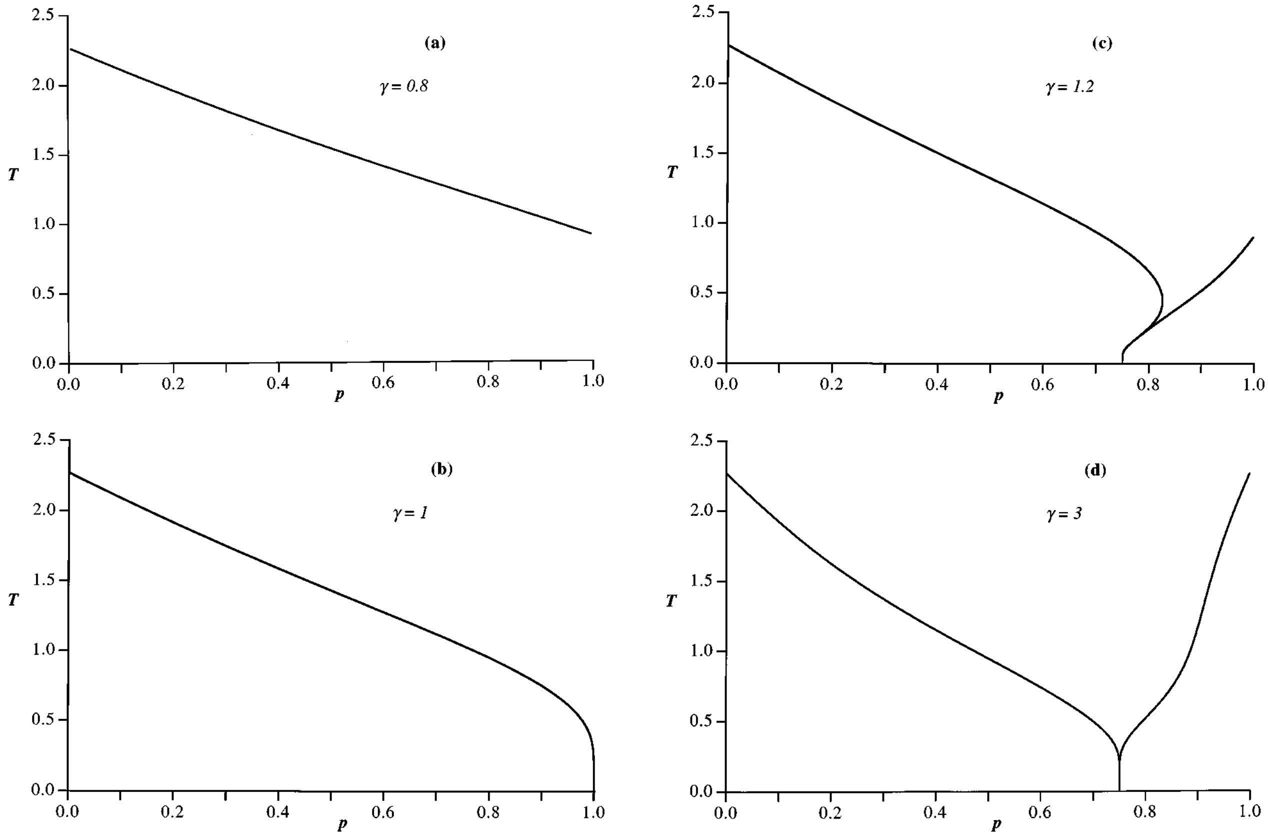

FIG. 3. p-T phase diagram at different γ: (a) $\gamma=0.8$; (b) $\gamma=1$; (c) $\gamma=1.2$; (d) $\gamma=3$.

always zero for $\gamma<1$ and $S_0 \geqslant 0$ for $\gamma=1$. In the case $\gamma>1$ the entropy is negative for $p<\tilde{p}$, where $\tilde{p}<\frac{3}{4}$ and becomes positive for $p>\tilde{p}$ [$S_0(p=\frac{3}{4}) \simeq 0.01$]. It should be kept in mind that the negative entropy at very low temperature is an artifact of the annealed approximation which may become very bad when $T \to 0$. From a technical point of view this is a consequence of the fact that one does not take into account the entropic contribution due to the different realizations of the disorder.

IV. TRANSITIONS

Let us stress again that formulas (3.3)–(3.6) represent the solution of the model, and, in principle, all the information about it can be derived from them. Fortunately, even if we are unable to give an explicit expression of b^* starting from Eq. (3.6), we can easily obtain some analytic results. For instance, in this section we find the conditions that yield to a second-order phase transition.

Looking carefully at Eqs. (3.3)–(3.6), one can realize that the mechanism of the usual Onsager transition is preserved: the discontinuity occurs when b^*, the zero of Eq. (3.6), nullifies also the argument of the square root in Eq. (3.4), i.e., when

$$t(q,b^*)=1 \tag{4.1}$$

[the case $t(q,b^*)=-1$ is not possible since $t(q,b) \geqslant 1$ for $\forall q$ and $\forall b$]. In other terms, one has to find out the solution b^* of a system of two equations, (3.6) and (4.1). Obviously this solution can exist only for certain values of β, γ, and p.

A direct inspection of Eq. (3.5) shows that Eq. (4.1) can be satisfied only for the specific choices $q=0$ or $q=\pi$, and it determines the existence of two distinct transition lines in the p-T phase diagram at fixed γ (see Fig. 3). The first line ends on the $p=0$ (pure Ising model) axis at the Onsager critical temperature, so that in the following we will refer to this transition line as the ferromagnetic one. The second line exists only for $\gamma \geqslant 1$ and it ends on the $p=1$ axis in correspondence with the critical temperature separating the antiferromagnetic phase from the paramagnetic one; for this reason we will call it the antiferromagnetic line.

After some trivial algebra Eq. (4.1) reduces to

$$\sinh(2\beta+b^*)=\pm 2\,\frac{\cosh 2\beta}{\sinh^2 2\beta}, \tag{4.2}$$

where the sign + corresponds to the ferromagnetic line ($q=0$), and the sign − to the antiferromagnetic one ($q=\pi$).

In the limit case $p=0$ ($b^*=2\beta$), Eq. (4.2) recovers the well-known result $\sinh(2\beta)=1$, while in the other limit case $p=1$ ($b^*=-2\beta\gamma$) the transition is present when $\sinh[2\beta(1-\gamma)]=-2\cosh 2\beta/\sinh^2 2\beta$, i.e., at finite temperature when $\gamma>1$ and at zero temperature for the "fully frustrated model" ($\gamma=1$).

From a practical point of view, in order to compute numerically the transition lines which are shown in Fig. 3, it is convenient to solve Eq. (4.1) with respect to b^*,

$$b^*=\pm \operatorname{arcsinh}\left(2\,\frac{\cosh 2\beta}{\sinh^2 2\beta}\right)-2\beta. \tag{4.3}$$

Then, keeping γ fixed and substituting b^* into Eq. (3.6), one obtains the two transition lines $p(T)$, which can be easily computed by standard numerical algorithms. The two previous equations for the phase boundaries could also be rewritten in a simpler form in terms of the dual couplings following [25] where similar expressions are found.

V. PHASES

In the preceding section we have seen that Eq. (4.2) gives the known critical temperatures of the nonrandom models ($p=0$ or 1). Other preliminary information about the behavior of the model comes from the observation that the right hand side of Eq. (4.2) goes to zero in the limit $T\to 0$, so that the ferromagnetic and the antiferromagnetic lines must coincide when they end on the $T=0$ axis. Studying the leading terms of Eq. (3.6) close to $T=0$, one finds that this coinciding point is at $p=1$ for $\gamma=1$ and at $p=\frac{3}{4}$ for $\gamma>1$.

The full description of the different behaviors can be derived computing the transition lines in the p-T phase diagram at varying γ, as seen in the preceding section. The following four scenarios listed in Fig. 3 are obtained.

For $\gamma<1$ [see Fig. 3(a), where $\gamma=0.8$] only the ferromagnetic line is present, separating two well-known phases: a ferromagnetic phase at low temperature, and a paramagnetic phase at high temperature, exactly as for the Onsager nonrandom model. In fact the antiferromagnetic random couplings are too weak with respect to the ferromagnetic ones, so that they are not able to change the structure of the phases of the pure model.

When $\gamma=1$ [Fig. 3(b)] the scenario is quite similar to the previous one, apart from the fact that the ferromagnetic line reaches the axis $T=0$ at $p=1$, in correspondence with the $T=0$ transition of the fully frustrated model. Notice that the antiferromagnetic transition line is still absent. In fact, the antiferromagnetic couplings have the same strength as the others but, for any $p<1$, their number is lower than the number of horizontal ferromagnetic couplings so that the ferromagnetic order prevails at low temperature.

The most interesting situation corresponds to the choice $1<\gamma<2$ [Fig. 3(c), where $\gamma=1.2$]. First of all notice that both the transition lines are present. The first line starts on the $p=0$ axis and it ends at $p=\frac{3}{4}$ on the $T=0$ axis and it delimits the low temperature ferromagnetic region. The second line starts on the $T=0$ axis at $p=\frac{3}{4}$ ending on the $p=1$ axis, and it delimits the antiferromagnetic phase. Outside these two regions there is a nonmagnetic phase, but notice that this one has a narrow tongue dividing the magnetic regions and reaching the $T=0$ axis at $p=\frac{3}{4}$. As a consequence, if one fixes the probability between $\frac{3}{4}<p<\frac{3}{4}+\delta p(\gamma)$, where $\delta p(\gamma)$ is a small but finite number depending on γ, one can observe three different second-order phase transitions varying the temperature T. The transitions separate four phases; starting from low temperatures, the first is ferromagnetic, the third is antiferromagnetic, the fourth is an ordinary paramagnetic phase while the second is a low temperature paramagnetic phase. In Fig. 4 is shown the specific heat C as a function of T at fixed $\gamma=1.2$ and $p=0.82$, computed starting from the numerical solution of Eqs. (3.3) and (3.6). In this case the specific heat C exhibits three distinct peaks next to $T\simeq 0.299$, $T\simeq 0.349$, and $T\simeq 0.545$. Indeed we need to magnify the picture since it is necessary to compute Eq. (3.3) with a great precision in order to show a certain growth of C around its discontinuity.

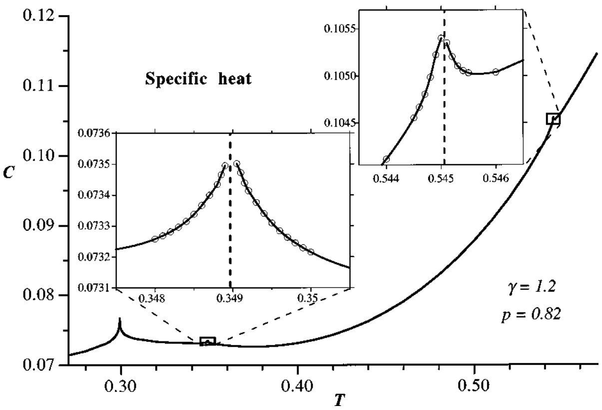

FIG. 4. Specific heat C as a function of T for $\gamma=1.2$ and $p=0.82$. The first peak, located at $T\simeq 0.299$, is referred to an antiferromagnetic transition, while the others, magnified in insets ($T\simeq 0.349$ and $T\simeq 0.545$), to ferromagnetic transitions. The circles in insets represent our numerical data.

The appearance of a low temperature paramagnetic phase between the antiferromagnetic and the ferromagnetic ones represents an interesting peculiarity of this model. In particular, it happens at relatively low temperature and with an extremely narrow width. These are the main features that persuade us to guess a glassy nature for this reentrant paramagnetic phase. Moreover, in our constrained annealed model, as seen at the end of Sec. III, the region at low temperature with an unphysical solution (negative zero temperature entropy S_0) does not reach the critical transition point ($p=\frac{3}{4}$, $T=0$) where the reentrant paramagnetic phase ends. For this reason, we can assume that the main qualitative features of the annealed approximation remain unchanged in the quenched model.

The description of the different scenarios is completed with the case $\gamma>2$ [Fig. 3(d), where $\gamma=3$]. The structure of the phase diagram is similar to the previous one with the difference that the narrow tongue between the ferro and the antiferro phases is suppressed. As a consequence for $p<\frac{3}{4}$ we only have the ferro and the para phases while for $p>\frac{3}{4}$ we only have the antiferro and the para phases. When $\gamma\to\infty$ the temperature of end point on the $p=1$ axis goes to infinite.

Before ending this section we would like to stress that both numerical evidence and a direct inspection of analytic expressions suggest that the logarithmic divergence at the transition lines remains unchanged with respect to the pure Ising system, so that the model cannot be described in the general framework of the Fisher renormalization of critical exponents [26].

VI. CONCLUSIONS

The surprising feature of our model is that for some choices of the parameters γ and p the magnetic phases are separated by a low temperature paramagnetic phase. We do not expect any long distance magnetic order in this phase, i.e., $\overline{\langle\sigma_{i,j}\sigma_{i,j+k}\rangle}=\overline{\langle\sigma_{i,j}\sigma_{i+k,j}\rangle}=0$ in the limit $k\to\infty$ but we expect that $\overline{\langle\sigma_{i,j}\sigma_{i,j+k}\rangle^2}>0$ and $\overline{\langle\sigma_{i,j}\sigma_{i+k,j}\rangle^2}>0$ in the same limit. Our proposal is that these last two quantities properly characterize the reentrant paramagnetic phase (even if they should not vanish in the high temperature paramagnetic phase). Unfortunately, it is well known that in two-dimensional models the computation of long distance correlation is a difficult task and, in our case, the computation also involves averages over the disorder, making the situation even more complicated. We think that some work can be done in this direction but it will demand much technical effort so that our claim that the reentrant paramagnetic phase is somehow a glassy phase is, at this point, more a conjecture than an established fact.

Two more questions remain to be answered. The first is the most relevant: what is the role of the annealed approximation in the qualitative features of the phase diagram? Or better, is the new phase a mere consequence of the annealed approximation? In this case, our fixed frustration model would have an interest in itself but it would not be a good approximation of the quenched one. We think that this question can only be answered by direct Monte Carlo simulation.

The second question is, in this model the frustrated plaquettes appear only in couples, what is the role of this special correlation? To be more specific, would a model in which all vertical bonds equal unity while the horizontal are independent random variables which take two possible values of opposite sign have the same qualitative behavior? We cannot approximate such a model by our fixed frustration technique so that this question can also only be answered by Monte Carlo simulation; nevertheless, we are convinced that the answer should be positive. In fact, the special nature of correlation between plaquettes is short ranged and there is no reason why it should affect the long range behavior of the system.

In conclusion, we would like to stress that in spite of the very partial results contained in this paper and of the many unsolved questions this work sheds some light on the very important point of the existence of a finite temperature disordered phase for $d=2$ frustrated systems. In fact, in the most restrictive interpretation of our result we can still affirm that the fixed frustration annealed $d=2$ model has a low temperature glassylike phase, while in the most generous one we can say that a true glassy phase exists for quenched random $d=2$ systems.

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